2024

STATISTICS — MINOR

Paper : MN-1

(Descriptive Statistics-I and Probability-I)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** questions : 2×5

   (a) Explain, with an example, what you mean by an attribute.

   (b) Distinguish between a discrete and a continuous variable.

   (c) If $y = 10^5 - 2x$ and the variance of x is 4, what is the standard deviation of y?

   (d) Give an example where the harmonic mean is a suitable measure of central tendency.

   (e) What do you mean by skewness of a frequency distribution?

   (f) For three events A, B and C, symbolically express the events of occurrence of exactly one of them and at least one of them.

   (g) Define conditional probability.

   (h) Write down two major drawbacks of the classical definition of probability.

2. Answer ***any four*** questions :

   (a) Define arithmetic mean of a frequency distribution. Suppose a variable takes the values 1, 2, ..., n with corresponding frequencies $1^2, 2^2, \ldots, n^2$. Find the arithmetic mean. 2+3

   (b) Define lower and upper quartiles. Indicate how one can find them from a single ogive. 3+2

   (c) Prove that mean deviation is least when measured about the median. 5

   (d) Define mutually exclusive and independent events. Discuss whether two mutually exclusive events can be independent or not. 3+2

   (e) State and prove the theorem of total probability. 2+3

   (f) Define pairwise and mutually independent events. Illustrate with example. 5

3. Answer ***any three*** questions :

   (a) (i) Define geometric mean of a set of positive observations. Show that it cannot exceed the arithmetic mean. When will these two be equal?

   (ii) State and prove any two properties of the geometric mean. (2+6+1)+6

(b) (i) Define standard deviation of a variable. Show that it is independent of the change of base but is dependent on the change of scale.

(ii) State and prove one of the inequalities relating the standard deviation of a set of observations to its range.

(iii) Find the standard deviation of the first n natural numbers. (2+3)+5+5

(c) (i) What are ordinary and central moments of a set of observations? Express r-th order central moment in terms of r-th and lower order ordinary moments.

(ii) Define the b_1 and b_2 measures. State and prove any inequality between them. (3+4)+(3+5)

(d) (i) Distinguish, with example, between elementary and composite events.

(ii) If $P(B \cap C) > 0$, then show that $P(A \cap B \mid C) = P(B \backslash C)\, P(A \backslash B \cap C)$.

(iii) If A, B, C and D are independent events, show that $A \cup B$ and $C \cup D$ are also independent.

(iv) State and prove the theorem of total probability for three events. 3+3+4+5

(e) (i) State and prove Bayes' Theorem.

(ii) Three urns contain, respectively, 4 red and 6 green balls; 5 red and 5 green balls; 6 red and 4 green balls. One of the urns is selected at random and a ball is drawn from the selected urn. If it is found to be red, what is the probability that the 2nd urn was selected? (2+6)+7